LEVELS OF ABSTRACTION IN SCHOOL ARITHMETIC

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In my article of A critique of the structure of U.S. elementary school mathematics (Ma, 2013), I discussed two organization types of elementary school mathematics. One has a "core-subject structure," and the other "a strands structure." I used the Chinese elementary school math standards before 2001 and the U. S. NCTM Standards as the examples to illustrate a comparison between them (Figure 1):

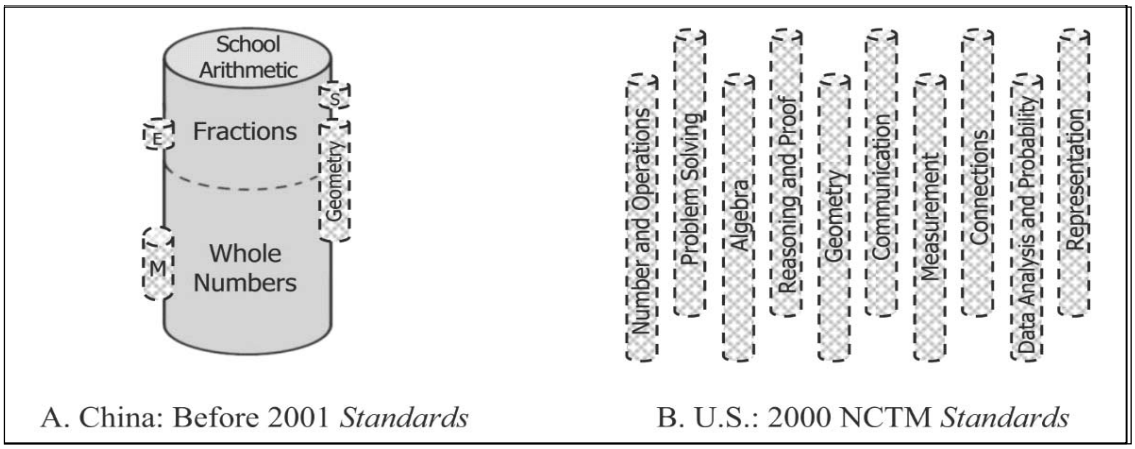

Figure 1. Two organizations of elementary school mathematics.

Example A has a "core-subject structure." The large gray cylinder in the center represents school arithmetic. Its solid outline indicates that it is a "self-contained subject." School arithmetic consists of two parts: whole numbers and fractions. Knowledge of whole numbers is the foundation upon which knowledge of fractions is built. The smaller cylinders represent the four other components of elementary mathematics, shown according to the order in which they appear in instruction. These are: measurement (M), elementary geometry, simple equations (E), and simple statistics (S). The core subject of elementary mathematics is what I call "school arithmetic." The subject of school arithmetic was constructed following the model of Euclid's *The Elements*. Although it took several decades to be comprehensively developed, its feature of being self-contained never changed. That feature ensures the consistency of elementary mathematics contents with school arithmetic as the core.

Example B has a "strands structure." Its components are juxtaposed, but not connected. Each of the ten

cylinders represents one standard in *Principles and Standards for School Mathematics.* No self-contained subject is shown. This type of structure has existed in the U.S. for almost fifty years, since the beginning of the 1960s. The number and the components in a "Strands Structure" can be frequently changed and replaced, according to the different visions of the education policy makers. In this way, the consistency becomes a "luxury" hard to attain.

In the article The Theory of School Arithmetic: Whole Numbers (Ma & Kessel, 2018), Kessel and I pointed out that the idea of "unit one" is the fundamental concept on which the subject of school arithmetic is built on. We also addressed on several stages that the concept of "unit one" evolves in whole numbers, and how they may inspire students' abstractive thinking step by step.

In this speech I would like to expand the issue into fractions. A more detailed description of the evolution of the concept of "unit one" in school arithmetic, from concrete to abstract, from simple to sophisticated, will be discussed.

1) One-digit numbers. Addition and subtraction exclusively with one-digit numbers;
2) Multi-digit numbers. Addition and subtraction with whole numbers;
3) Multiplication and division with whole numbers;
4) The four fundamental operations of whole numbers;
5) Fractions. Addition and subtraction with fractions;
6) Multiplication with fractions;
7) Division with fractions;
8) The four fundamental operations of fractions.

The Fig. 2 presents the eight levels of abstraction levels of the concept of unit one in school arithmetic:

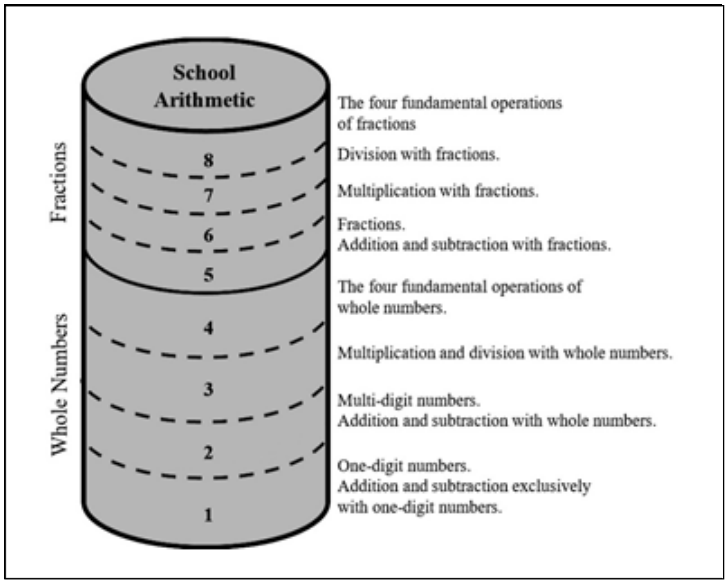

Figure 2. Eight abstraction levels of the concept of unit one in school arithmetic

I will use word problems to illustrate these eight levels of abstraction.

References

Ma, L. (2013). A critique of the structure of U.S. elementary school mathematics. *Notices of the American Mathematical Society*, *60*(10), 1282–1296, http://www.ams.org/notices/201310/fea-ma.pdf.

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